B.COM (PART-III)HONOURS IN ACCOUNTING & FINANCE

SUBJECT CODE:3CH1

SUBJECT: BUSINESS ECONOMICS AND QUANTITATIVE TECHNIQUES

TOPIC-LIMITS AND CONTINUTY

LIMITS AND CONTINUTY

LIMITS

The concept of limits is useful in developing some mathematical techniques and also in analyzing various economic problems in economics. For example, the rate of interest 'i' can never fall to zero even if the quantity of capital available in the economy is very large. There is a minimum below which 'i' cannot fall. Let this minimum be 2%, then if 'K' stands for capital and 'C' is a positive constant we can write the function as

$$i = 2 + \frac{C}{K}$$

If K is small 'i' is large

If K is large 'i' is small

If K is very large (infinite) 'i' will never be less than 2. Therefore the limit 'i' as K increase is 2.

It can be written as $\underset{K \to \infty}{\text{Lt}}\ i = 2$

A function f(x) is said to approach the limit 'l' as 'x' approach to 'a' if the difference between f(x) and 'l' can be made as small as possible by taking 'x' sufficiently nearer to 'a'.

Example 1: Limit x^2 as x approach to 4 is 16.

This can be expressed as

$$\underset{x \to 4}{\text{Lt}}\ x^2 = 16$$

i.e., the difference between x^2 and 16 can be made as small as possible by taking 'x' sufficiently nearer to 4.

Ex. 2: If $f(x) = 5x + 10$ find $\underset{x \to 0}{\text{Lt}}\ f(x)$

Substituting $x = 0$ $\qquad \underset{x \to 0}{\text{Lt}}\ f(x) = 10$

Ex. 3: Find $\underset{x \to 3}{\text{Lt}}\ x^2 + 10x + 5$

Ans: $\qquad \underset{x \to 3}{\text{Lt}}\ (x^2 + 10x + 5)$

$$= \underset{x \to 3}{\text{Lt}}\ x^2 + 10 \underset{x \to 3}{\text{Lt}} x + \underset{x \to 3}{\text{Lt}} 5$$

$$= \underline{\underline{9 + 30 + 5 = 44}}$$

Theorems on Limit of Function

1. If 'a' is a constant

$$\lim_{x \to K} a = a$$

2. If a, b are constants

$$\lim_{x \to K} (ax + b) = ak + b$$

3. If $\lim_{x \to K} f(x) = l$, $\lim_{x \to K} g(x) = m$ then

(i) $$\lim_{x \to K} [f(x) + g(x)] = l + m$$

(ii) $$\lim_{x \to K} [f(x) . g(x)] = l \text{ x } m$$

(iii) $$\lim_{x \to K} \frac{f(x)}{g(x)} = \frac{l}{m}, \quad m \neq 0$$

(iv) $$\lim_{x \to K} \sqrt[n]{f(x)} = \sqrt[n]{l}$$

(v) $$\lim_{x \to K} f(x) = l, \quad \lim_{x \to K} g(x) = 0$$

$$\lim_{x \to K} f(x) . g(x) = 0$$

Ex. 3: Find $\lim_{x \to 2} \frac{4x^2-5x+10}{10x+5}$

Ans: $$\lim_{x \to 2} \frac{4x^2-5x+10}{10x+5} = \frac{4\times2^2-5\times2+10}{10\times2+5}$$

$$= \frac{4\times4-10+10}{20+5} = \underline{\underline{\frac{16}{25}}}$$

Ex. 4: Find $\lim_{x \to 3} (x+2)^2 + 5$

Ans. $\underset{x \to 3}{\text{Lt}} \ (x+2)^2 + 5 = (3+2)^2 + 5$

$= 5^2 + 5 = 25 + 5 = \underline{\underline{30}}$

Indeterminate Form

If the value of the function becomes indeterminate like $\frac{0}{0}$, $\frac{\infty}{\infty}$ on substituting the value of the variables, mere substitution method may not be feasible. In such case factorization of the function may remove the difficulties.

For example: $\underset{x \to 3}{\text{Lt}} = \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{9-9}{3-3} = \frac{0}{0}$

$\therefore$ write $\frac{x^2-9}{x-3}$ as $\frac{x^2-3^2}{x-3}$

$= \frac{(x+3)(x-3)}{x-3} = x + 3 = \underline{\underline{6}}$

[Note $a^2 - b^2 = (a+b)(a-b)$]

Ex. 2: $\underset{x \to \infty}{\text{Lt}} = \frac{x^2+1}{x^2-1} = \frac{\infty}{\infty}$

$= \underset{x \to \infty}{\text{Lt}} \frac{\frac{x^2}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$ [division of numerator and denominator by x^2]

$= \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$

Since $\frac{1}{x^2} \to 0$ as $x \to \infty$

We go $\frac{1}{1} = \underline{\underline{1}}$

Ex. 3: $\underset{x \to 3}{\text{Lt}} = \frac{x^2 - 4x+3}{x^2 + 2x-3}$

Factorizing the function we get

$$\frac{(x-3)(x-1)}{(x+3)(x-1)} = \frac{x-3}{x+3} = \frac{1-3}{1+3} = \frac{-2}{4} = \frac{-1}{2}$$

<u>Problems</u>

1. Find $\underset{x \to 2}{\text{Lt}} \frac{x^3 - 3x+2}{x^2 - 5x+6}$ [Ans: -1]
2. Find $\underset{x \to 1}{\text{Lt}} \frac{x^2-1}{x-1}$ [Ans: 2]
3. Find $\underset{x \to 2}{\text{Lt}} \frac{r^2 - 5}{2r^3+6}$ $\left[\text{Ans}: \frac{-1}{22}\right]$
4. Find $\underset{x \to 0}{\text{Lt}} \frac{5x^2 + 8x}{x}$ [Ans: 8]
5. Find $\underset{x \to \infty}{\text{Lt}} \frac{10x^2 + 5x}{5x^2}$ [Ans: 2]
6. Find $\underset{x \to 6}{\text{Lt}} \frac{x^2 - x-30}{x^2 - 4x-12}$ $\left[\text{Ans}: \frac{11}{8}\right]$

CONTINUITY

A function is said to be continuous in a particular region if it has determinate value (or a finite quantity) for every value of the variable in that region.

i.e., $\underset{x \to a}{Lt}$ f(x) may or may not be the same as f(a)

If $\underset{x \to a}{Lt}$ f(x) = f(a) a finite quantity, then f(x) is said to be a continuous function of x at x = a.

If a function is continuous over a range it is continuous at every point of the range and can be represented by a curve which has no gaps and no jumps in the range concerned. Therefore a function f(x) is said to be continuous at x = a if f(x) tends to f(a)from either side.

$$\underset{x \to a^-}{Lt} f(x) = \underset{x \to a^+}{Lt} f(x) = f(a)$$

Where x→a^+ refers to f (a + h) and x→a^- refers to f(a – h) as h→0.

Condition for continuity

1. F(a) exists
2. $\underset{x \to a}{Lt}$ f(x) = f(a), a finite quantity.

If the conditions are not satisfied for any value of x, f(x) is said to be discontinuous for that value of 'x'.

Eg: If f(x) $= \frac{1}{x-2}$ and $\underset{x \to a}{Lt}$ f(x) $= \frac{1}{a-2}$

If x → 2. f(x) does not have a finite value

∴ f(x) is continuous for any value of 'a' except 2.

Eg. 1: Check for continuity of function at x = 1

$$f(x) = \frac{x^2 + 3x - 4}{x-1}$$

Solution :

$$\underset{x \to 1}{Lt} f(x) = \underset{x \to 1}{Lt} \frac{x^2 + 3x - 4}{x-1}$$

$$\underset{x \to 1}{Lt} \frac{(x+4)(x-1)}{(x-1)} = x + 4 = 5$$

But f(1) does not exist

i.e., substituting 1 for x in f(x) function $\frac{1^2 + 3\times 1 - 4}{1-1}$

$$= \frac{0}{0} = \infty \text{ (indeterminate)}$$

Thus f(x) is not continuous at x =1

Eg. 2: Prove that the function $x^2 + 3x - 2$ is continuous for x = 2

Ans: $f(2) = 2^2 + 3 \times 2 - 2 = 8$ is finite.

$$\underset{x \to 2}{Lt} f(x) = \underset{h \to 0}{Lt} (2-h)^2 + 3(2-h) - 2$$

$$= 2^2 + 3 \times 2 - 2 = 8$$

$$\therefore \underset{x \to 2^-}{Lt} f(x) = \underset{h \to 2^+}{Lt} f(x) = f(2)$$

∴ f(x) is continuous at x = 2.

THANK YOU

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